

Exercise 1

Solve, using Fourier Transforms

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

for $0 < y < H$, $-\infty < x < \infty$ subject to the initial/boundary conditions

$$u(x, 0) = 0$$

$$\frac{\partial u}{\partial y}(x, H) + hu(x, H) = f(x)$$

Exercise 2

Solve, using Fourier Transforms

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

for $x < 0$, $-\infty < y < \infty$ subject to $u(0, y) = g(y)$.

Exercise 3

Solve

$$\frac{\partial u}{\partial t} + v_0 \cdot \nabla u = k \nabla^2 u$$

subject to the initial condition

$$u(x, y, 0) = f(x, y)$$

Show how the influence function is altered by the convection term $v_0 \cdot \nabla u$.

Exercise 4

Solve, via Fourier Transforms:

$$\frac{\partial u}{\partial t} = k_1 \frac{\partial^2 u}{\partial x^2} + k_2 \frac{\partial^2 u}{\partial y^2}$$

with the initial condition

$$u(x, y, 0) = f(x, y)$$

Exercise 5

Solve, via Fourier Transforms

$$\frac{\partial u}{\partial t} = k \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

with $x > 0$ and $y > 0$ and the initial condition

$$u(x, y, 0) = f(x, y)$$

and the bound conditions

$$u(0, y, t) = 0 \quad \frac{\partial u}{\partial y}(x, 0, t) = 0$$